

The effect of spin magnetization in the damping of electron plasma oscillations

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The effect of spin of particles in the propagation of plasma waves is studied using a semi-classical kinetic theory for a magnetized plasma. We focus in the simple damping effects for the electrostatic wave modes besides Landau damping. Without taking into account more quantum effects than spin contribution to Vlasov's equation, we show that spin produces a new damping or instability which is proportional to the zeroth order magnetization of the system. This correction depends on the electromagnetic part of the wave which is coupled with the spin vector.

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One of the most important physical results in the propagation of plasma waves which cannot be deduced by a fluid description is Landau damping [1]. This effect predicts that an electron plasma wave, in an collisionless plasma, suffers damping owing to the wave-particle interaction. Damping mechanism will depend on the velocity distribution of the particles which are often Maxwellian. The mathematical procedures to obtain Landau damping are very standard in kinetic theory. Thus, the formalism has been extended for other kind of electron interactions, for example thermal ion Landau damping effects [2], and Landau damped electron waves by photons [3] or neutrinos [4].

On the other hand, recently there have been a huge interest in the field of plasma physics for the dynamics of the spin of electrons, and how its quantum nature affects the different known modes of propagations [5–7] or other properties [8, 9]. These treatments are useful in, for example, astrophysical systems [10] or high-energy lasers [11]. Often these effects are important in high density, low temperature or strong magnetic fields conditions, but it has been shown that for some systems at high temperature the spin dynamics play a crucial role [12].

In a previous work [13], we present a first approach to calculate a correction to Landau damping due to spin. In this letter, we complete the analysis done in the previous work finding a full solution for the damping produced by the spin to electrostatic modes. We will show that this new damping is proportional to the spin magnetization of the plasma, and it depends on the electromagnetic part of the wave.

To obtain the correction to the Landau damping produced by the spin of the plasma constituents, we use the semi-classical kinetic theory constructed in Ref. [14] which start from the Pauli Hamiltonian. Here, the dynamics of the spin of particles is included in a Vlasov

equation for a generalized distribution function. Other quantum corrections, as Bohm potential, spin-spin interaction force and high order force terms in the spin evolution equation are neglected. Thus, Vlasov equation, for particles with velocity \mathbf{v} , spin vector \mathbf{s} and with distribution function $f = f(\mathbf{r}, \mathbf{v}, \mathbf{s}, t)$ is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \left[\frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{2\mu_e}{m\hbar} \nabla (\mathbf{s} \cdot \mathbf{B}) \right] \cdot \nabla_v f + \frac{2\mu_e}{\hbar} (\mathbf{s} \times \mathbf{B}) \cdot \nabla_s f = 0, \quad (1)$$

where $q = -e$ and m are the electron charge and mass respectively, $\mu_e = -ge\hbar/(4m)$ is the electron magnetic moment and $g \approx 2.002319$ is the electron spin factor. The Fermi-Dirac equilibrium distribution function $f_0(\mathbf{v}, \mathbf{s})$ is [14]

$$f_0(\mathbf{v}, \mathbf{s}) = \frac{B_0 \mu_e \tilde{f}_0(\mathbf{v})}{4\pi k_B T \sinh(B_0 \mu_e / k_B T)} \exp\left(\frac{2\mu_e \mathbf{s} \cdot \mathbf{B}_0}{\hbar k_B T}\right), \quad (2)$$

where T is the temperature, k_B is the Boltzmann constant, \mathbf{B}_0 is a background magnetic field ($B_0 = |\mathbf{B}_0|$) and \tilde{f}_0 is the classical Maxwellian distribution function

$$\tilde{f}_0(\mathbf{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right), \quad (3)$$

with $v = |\mathbf{v}|$. The distribution function (2) is normalized as $\int f_0 d\mathbf{v} d\mathbf{s} = 1$, where the integration is made over the three degree of freedom in velocity space and the two degree of freedom in spin space.

Now, we use the kinetic formalism (1) and a similar analysis of Ref. [13] to derive the dispersion relation for electron plasma oscillations in a magnetized plasma which interacts with the spin of the particles. The electric and magnetic fields will be perturbed in the form $\mathbf{E} = \mathbf{E}_1$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$ respectively. The terms with subscript 0 are the zeroth order equilibrium quantities, and the terms with subscript 1 are the first order perturbed quantities. The distribution function is perturbed as $f(\mathbf{r}, \mathbf{v}, \mathbf{s}, t) = f_0(\mathbf{v}, \mathbf{s}) + \hat{f}_1(\mathbf{r}, \mathbf{v}, \mathbf{s}, t)$ and we

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choose $\mathbf{B}_0 = B_0 \hat{z}$. The perturbed distribution \hat{f}_1 will have the form $\hat{f}_1(\mathbf{r}, \mathbf{v}, \mathbf{s}, t) = f_1(\mathbf{v}, \mathbf{s}) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$, with similar assumption for other perturbed quantities. Here, \mathbf{k} and ω are the wavenumber and the frequency of the wave.

Linearizing Eq. (1), with the velocity \mathbf{v} and spin \mathbf{s} as independent variables and following Ref. [13], we can find the perturbed distribution function as

$$f_1 = \frac{-i}{\omega - \mathbf{k} \cdot \mathbf{v}} \left(\frac{q}{m} \mathbf{E}_1 \cdot \nabla_v f_0 + \frac{2\mu_e}{m\hbar} \nabla(\mathbf{s} \cdot \mathbf{B}_1) \cdot \nabla_v f_0 + \frac{2\mu_e}{\hbar} (\mathbf{s} \times \mathbf{B}_1) \cdot \nabla_s f_0 \right), \quad (4)$$

because $\mathbf{v} \times \mathbf{B}_1 \cdot \nabla_v f_0 = 0$.

From here, we are going to focus in the study of spin correction to the Landau damping for electrostatic modes. Let us concentrate the charge density ρ of the

plasma which is given by $\rho = qn_0 \int f_1 d\mathbf{v} d\mathbf{s}$, where n_0 is the equilibrium density. Using the Maxwell equation $\mathbf{k} \times \mathbf{E}_1 = \omega \mathbf{B}_1$, the perturbed distribution function (4), and defining the quantities $\mathbf{E}_\perp = \mathbf{k} \times \mathbf{E}_1/k$ and $E_\parallel = \mathbf{k} \cdot \mathbf{E}_1/k$ with $k = |\mathbf{k}|$, the charge density becomes in

$$\rho = -iqn_0 \int d\mathbf{s} \int_{-\infty}^{\infty} \frac{d\mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}} \left(\frac{q}{mk} E_\parallel (\mathbf{k} \cdot \nabla_v f_0) + \frac{i2\mu_e k}{m\hbar\omega} (\mathbf{s} \cdot \mathbf{E}_\perp) \mathbf{k} \cdot \nabla_v f_0 + \frac{2\mu_e k}{\hbar\omega} (\mathbf{s} \times \mathbf{E}_\perp) \cdot \nabla_s f_0 \right). \quad (5)$$

This charge density must be used in the Poisson's equation $ikE_\parallel = 4\pi\rho$ to obtain the dispersion relation for electrostatic modes. In this way, the dispersion relation is

$$1 = \frac{\omega_p^2}{k^2} \int d\mathbf{s} \int_{-\infty}^{\infty} \frac{d\mathbf{v}}{\mathbf{k} \cdot \mathbf{v} - \omega} \left(1 + \frac{2i\mu_e k^2}{q\hbar\omega} \left(\frac{\mathbf{s} \cdot \mathbf{E}_\perp}{E_\parallel} \right) \right) \mathbf{k} \cdot \nabla_v f_0 + \frac{\omega_p^2}{k^2} \int d\mathbf{s} \int_{-\infty}^{\infty} \frac{d\mathbf{v}}{\mathbf{k} \cdot \mathbf{v} - \omega} \left(\frac{2\mu_e k^2 m}{q\hbar\omega} \right) \left(\frac{\mathbf{s} \times \mathbf{E}_\perp}{E_\parallel} \right) \cdot \nabla_s f_0, \quad (6)$$

where $\omega_p^2 = 4\pi e^2 n_0/m$ is the square of the plasma frequency. When the spin contribution is neglected ($\mu_e = 0$), we reobtain the classical dispersion relation for electrostatic modes.

To solve the dispersion relation (6), we need to evaluate the two integral involving the spin contribution. The integration in the two degree of freedom in spin space is done in spherical coordinates such that $d\mathbf{s} \equiv d\Omega_s = d(\cos\theta_s)d\phi_s$ where the subindex s is for spin coordinates. In the same sense, the spin vector will be $\mathbf{s} = -\hbar/2\hat{s} = -\hbar/2(\sin\theta_s \cos\phi_s \hat{x} + \sin\theta_s \sin\phi_s \hat{y} + \cos\theta_s \hat{z})$,

and $\nabla_s f_0 = 2\mu_e B_0 \sin\theta_s f_0/(\hbar k_B T) \hat{\theta}$. The choice on the spin orientation is to minimize the magnetic moment energy, which is consistent with paramagnetism [15]. On the other hand, the above integrations in velocity and spin space can be simplified introducing the one-dimensional distribution

$$F_0(u, \mathbf{s}) \equiv \int f_0 \delta\left(u - \frac{\mathbf{k} \cdot \mathbf{v}}{k}\right) d\mathbf{v}. \quad (7)$$

We can use (7) to rewrite the dispersion relation (6) as

$$1 = \frac{\omega_p^2}{k^2} \int d\Omega_s \int_{-\infty}^{\infty} \frac{du}{u - \omega/k} \left(1 - \frac{i\mu_e k^2}{q\omega} \left(\frac{\hat{s} \cdot \mathbf{E}_\perp}{E_\parallel} \right) \right) \frac{\partial F_0}{\partial u} - \frac{2\omega_p^2 \mu_e^2 m B_0}{q\hbar\omega k_B T} \int d\Omega_s \int_{-\infty}^{\infty} \frac{du \sin\theta_s F_0}{u - \omega/k} \frac{(\hat{s} \times \mathbf{E}_\perp) \cdot \hat{\theta}}{E_\parallel}. \quad (8)$$

The integrals in Eq. (8) must be evaluated as a contour integral considering the singularity at $u_\phi \equiv \omega/k$. This is the origin of classical Landau damping. We consider the case of large phase velocity u_ϕ and weak damping, where the pole lies near the real u axis. In this case, F_0 and $\partial F_0/\partial u$ are both small near u_ϕ . Neglecting the thermal correction to the real part of the frequency, the first two

integrals are given by [16]

$$\int d\Omega_s \int_{-\infty}^{\infty} \frac{du}{u - \omega/k} \frac{\partial F_0}{\partial u} \simeq \frac{k^2}{\omega^2} + i\pi \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u=u_\phi}, \quad (9)$$

$$\int d\Omega_s \int_{-\infty}^{\infty} \frac{du}{u - \omega/k} \frac{\hat{s} \cdot \mathbf{E}_{\perp}}{E_{\parallel}} \frac{\partial F_0}{\partial u} \simeq -\chi\eta(\alpha) \left(\frac{k^2}{\omega^2} + i\pi \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u=u_{\phi}} \right), \quad (10)$$

where $\chi = \hat{z} \cdot \mathbf{E}_{\perp}/E_{\parallel}$, $\alpha = \mu_e B_0/k_B T$ and $\eta(x) = \coth(x) - 1/x$ is the Langevin function. \tilde{F}_0 comes from (7) and it is defined as

$$\tilde{F}_0(u) \equiv \int \tilde{f}_0 \delta \left(u - \frac{\mathbf{k} \cdot \mathbf{v}}{k} \right) d\mathbf{v}. \quad (11)$$

The third integral in dispersion relation (8) vanish due the only relevant spin contribution is anti parallel to the background magnetic field. Then, the dispersion relation (8) becomes

$$1 = \left(\frac{\omega_p^2}{\omega^2} + \frac{i\pi\omega_p^2}{k^2} \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u_{\phi}} \right) \left(1 + \frac{i\mu_e k^2}{q\omega} \chi\eta(\alpha) \right). \quad (12)$$

We seek a frequency which has a real and an imaginary part given by $\omega = \omega_r + i\omega_i$ such that $\omega_i \ll \omega_r$. Using this in Eq.(12), and solving for the real and imaginary parts, we can obtain the frequency for the electrostatic modes

$$\omega = \omega_r \left(1 + \frac{i\pi\omega_r^2}{2k^2} \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u_{\phi}} \right) + \frac{ik^2 M_0 \chi}{2n_0 q}, \quad (13)$$

where, neglecting terms of order $(\partial \tilde{F}/\partial u)|_{u_{\phi}}^2$, the real part of the frequency is given by

$$\omega_r = \omega_p \left(1 - \frac{M_0 \chi \pi \omega_p}{2qn_0} \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u_{\phi}} \right), \quad (14)$$

and $M_0 = n_0 \mu_e \eta(\alpha)$ is value of the spin magnetization of the system. This is because the distribution f_0 of Eq. (2), gives the zeroth order magnetization of the system $\mathbf{M}_0 = (2\mu_e n_0/\hbar) \int \mathbf{s} f_0 d\mathbf{v} ds = M_0 \hat{z}$ [14, 17]. Thus, using Eqs. (13) and (14) the imaginary part of the frequency is

$$\omega_i = \frac{\pi\omega_p^3}{2k^2} \frac{\partial \tilde{F}_0}{\partial u} \Big|_{u_{\phi}} + \frac{k^2 M_0 \chi}{2n_0 q}, \quad (15)$$

From (15) we note that there is a correction in the imaginary part of the frequency which appears due to the magnetization of the plasma due to spin of their constituents. This correction depends on the electromagnetic part of the waves, which is coupled with spin vector of each particle. As we discuss in our previous work [13], the interaction of spin with the perturbed magnetic field in a magnetized plasma is the responsible of this contribution to the damping of an electrostatic mode. Moreover, in the present analysis we show that the energy

transfer between waves and particles depends on the shape of the equilibrium function (classical Landau damping) and also on the spin magnetization of the plasma.

The spin damping correction of Eq. (15) depends on the ratio of the magnetization and the charge of each particle, i.e, $M_0/n_0 q = \hbar g \eta(\alpha)/4m$. It is expected that spin effects will be important at low temperatures, high densities and huge magnetic fields, and due to the dependences of η on the α parameter we can see that, in fact, the spin correction to the Landau damping is higher for high values of density and background magnetic field, and for low temperatures. However, this spin damping is proportional to \hbar and the main effect is the classical Landau damping.

Besides, we can see that the correction to the damping is proportional to χ , which depends on the transversal part of the electromagnetic wave. If $\chi > 0$ in the case of an electron plasma, $\eta(\alpha) < 0$ and the correction is a damping. Also, for the same electron plasma, if $\chi < 0$ the correction is an instability. When the wave has no electromagnetic transverse component $|\mathbf{E}_{\perp}| = 0$, then $\chi = 0$ and there are no spin correction to the damping. The exact value of this coefficient should be obtained solving the dynamical Maxwell equations with the current density $\mathbf{j} = qn_0 \int \mathbf{v} f_1 d\mathbf{v} ds$. The complete implications of the value of χ is being studied.

On the other hand, from Eq. (14) we note that there is a correction in the frequency of plasma oscillations. This corrections depends on the magnetization of the plasma and also on the shape of the distribution function. As spin corrections are proportional to \hbar and the derivative of the distribution function is small, as in the case of the imaginary part, the correction is small and the frequency of electron waves is near ω_p as expected.

In conclusion, we have shown that the incorporation of spin to kinetic theory in a magnetized plasma produces corrections to the classical Landau damping for electrostatic waves that depends on the magnetization of the system, and is due to the coupling between the spin vector and the electromagnetic part of the wave. In other words, in addition to the Landau mechanism to transfer energy from waves to particles, the inclusion of spin allows the energy transfer through the quantum interaction between spin and magnetic fields. However, the corrections are of \hbar order and, in the case of electron plasma and mawellian distribution functions, the electrostatic wave will show an evolution similiar to its classical dynamics when the spin is not included. In addition we shown that spin contribution also introduces a correction to the frequency of plasma oscillations which is also proportional to \hbar and it is due to the magnetization of the plasma.

All of these results show the importance of the kinetic theory of plasmas. The same formalism can be used to introduce other quantum contributions to classical plasma physics and derive new corrections and effects for strongly coupled plasmas, as well as plasmas in presence of large magnetic fields when quantum effects are

relevant.

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- [1] L. D. Landau, J. Phys. U.S.S.R. **10**, 25 (1946).
 - [2] R. J. Goldston, and P. H. Rutherford, *Introduction to Plasma Physics*, Institute of Physics Publishing, Bristol and Philadelphia (1995).
 - [3] R. Bingham, J. T. Mendonça, and J. M. Dawson, Phys. Rev. Lett. **78**, 247 (1997).
 - [4] L. O. Silva, R. Bingham, J. M. Dawson, J. T. Mendonça, and P. K. Shukla, Phys. Lett. A **270**, 265 (2000).
 - [5] P. K. Shukla, and L. Stenflo, Phys. Lett. A **357**, 229 (2006).
 - [6] A. P. Misra, Phys. Plasmas **14**, 064501 (2007).
 - [7] P. K. Shukla, Phys. Lett. A **369**, 312 (2007).
 - [8] A. P. Misra, and N. K. Gosh, Phys. Lett. A **372**, 6412 (2008).
 - [9] M. Marklund, B. Eliasson, and P. K. Shukla, Phys. Rev. E **76**, 067401 (2007).
 - [10] M. G. Baring, and A. K. Harding, Astrophys. J. **547**, 929 (2001).
 - [11] D. Kremp, Th. Bornath, M. Bonitz, and M. Schlages, Phys. Rev. E **60**, 4725 (1999).
 - [12] G. Brodin, M. Marklund, and G. Manfredi, Phys. Rev. Lett. **100**, 175001 (2008).
 - [13] F. A. Asenjo, Phys. Lett. A **373**, 48 (2009).
 - [14] G. Brodin, M. Marklund, J. Zamanian, Å. Ericsson, and P. L. Mana, Phys. Rev. Lett. **101**, 245002 (2008).
 - [15] G. Brodin and M. Marklund, New J. Phys. **9**, 277 (2007).
 - [16] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, Plenum Press, New York (1984).
 - [17] M. Marklund, and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).